Understanding Non-linear hydrodynamic response in HI collisions via Event-Plane correlations

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Introduction and motivation

- Understanding the $v_n$ gives understanding of the nature of initial geometry and fluctuations in it.
- Complementary information can be obtained by studying correlations between the phases $\Phi_n$ of the $v_n$.

Singles: \[ \frac{dN}{d\phi} \propto 1 + \sum_n 2v_n \cos n(\phi - \Phi_n) \]

$(\Phi_n - \Phi_m)$ correlations
Origin of the event plane correlations

Representation of flow vector: \( \vec{v}_n \equiv (v_n \cos n\Phi_n, v_n \sin n\Phi_n) \equiv v_n e^{-in\Phi_n} \)

Hydro response is linear for \( v_2 \) and \( v_3 \): \( v_n \propto \epsilon_n \) and \( \Phi_n \approx \Phi_n^* \) i.e.
\[
v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}
\]

Non-linear terms possible for higher \( n \)

\[
v_4 e^{-i4\Phi_4} = \alpha_4 \epsilon_4 e^{-i4\Phi_4^*} + \alpha_{2,4} \left( \epsilon_2 e^{-i2\Phi_2^*} \right)^2 + ...
\]

Hydrodynamic response to eccentricities

\[
= \alpha_4 \epsilon_4 e^{-i4\Phi_4^*} + \beta_{2,4} v_2^2 e^{-i4\Phi_2} + ...
\]

Similarly correlations can occur between three planes of different orders:

\[
v_5 e^{-i5\Phi_5} = \alpha_5 \epsilon_5 e^{-i5\Phi_5^*} + \alpha_{2,3,5} \epsilon_2 e^{-i2\Phi_2^*} \epsilon_3 e^{-i3\Phi_3^*} + ... \]

\[
= \alpha_5 \epsilon_5 e^{-i5\Phi_5^*} + \beta_{2,3,5} v_2 v_3 e^{-i(2\Phi_2+3\Phi_3)} + ...
\]

\[\text{arXiv:1111.6538}\]
Quantifying the two-plane correlations

- The correlations are entirely described by the differential distribution:

\[
\frac{dN_{\text{events}}}{d(k(\Phi_n - \Phi_m))} : k = \text{LCM}(m,n)
\]

- The multiplication by the *Lowest common multiple*, ‘k’ removes the \(n/m\)-fold ambiguity in \(\Phi_m/\Phi_n\).

- The distribution can be expanded as a Fourier series.
  - The Fourier coefficients \(V_{j,n,m}^j\) quantify the strength of the correlation.

\[
\frac{dN_{\text{events}}}{d(k(\Phi_n - \Phi_m))} = 1 + 2\sum_{j=1}^{\infty} V_{n,m}^j \cos(jk(\Phi_n - \Phi_m))
\]

\[V_{n,m}^j = \langle \cos(jk(\Phi_n - \Phi_m))\rangle\]

- Observables in general:

\[
\langle \cos(c_1 \Phi_1 + 2c_2 \Phi_2 + \ldots + lc_l \Phi_l)\rangle
\]

\[c_1 + 2c_2 + \ldots + lc_l = 0\]
Accounting for detector resolution

True planes: $\Phi_n$

Measured planes: $\Psi_n$ (different than true planes due to finite detector resolution)

Measure correlation between EP, followed by a simple resolution correction.

Desired correlator

$$\langle \cos k(\Phi_n - \Phi_m) \rangle = \frac{\langle \cos k(\Psi_n - \Psi_m) \rangle}{\text{Res}\{k\Psi_n\}\text{Res}\{k\Psi_m\}}$$

Resolution for individual planes

$$\text{Res}\{k\Psi_n\} = \sqrt{\langle \cos^2(k\Psi_n - k\Phi_n) \rangle}$$

Observed correlator

Three plane correlation

$$\langle \cos(nc_n \Phi_n + mc_m \Phi_m + lc_l \Phi_l) \rangle = \frac{\langle \cos(nc_n \Psi_n + mc_m \Psi_m + lc_l \Psi_l) \rangle}{\text{Res}\{nc_n \Psi_n\}\text{Res}\{mc_m \Psi_m\}\text{Res}\{lc_l \Psi_l\}}$$
List of correlators measured

Two plane

\begin{align*}
\langle \cos 4(\Phi_2 - \Phi_4) \rangle \\
\langle \cos 8(\Phi_2 - \Phi_4) \rangle \\
\langle \cos 12(\Phi_2 - \Phi_4) \rangle \\
\langle \cos 6(\Phi_2 - \Phi_3) \rangle \\
\langle \cos 6(\Phi_2 - \Phi_6) \rangle \\
\langle \cos 6(\Phi_3 - \Phi_6) \rangle \\
\langle \cos 12(\Phi_3 - \Phi_4) \rangle \\
\langle \cos 10(\Phi_2 - \Phi_5) \rangle
\end{align*}

Three plane

\begin{align*}
\text{“2-3-5”} & \quad \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \\
& \quad \langle \cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) \rangle \\
\text{“2-4-6”} & \quad \langle \cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6) \rangle \\
& \quad \langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle \\
\text{“2-3-4”} & \quad \langle \cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4) \rangle \\
& \quad \langle \cos(-10\Phi_2 + 6\Phi_3 + 4\Phi_4) \rangle
\end{align*}
Weighted correlations: Scalar-Product Method

- Scalar product method (weighted correlations) is an improvement of the EP method that takes into consideration Event-by-Event flow fluctuations.

- Each event is weighted by the flow vector magnitude in that event:
  \[ \bar{q}_n = (q_n \cos n\Phi_n, q_n \sin n\Phi_n) \]

- The weighted EP correlations are defined as:
  \[ \left\langle \cos(c_1\Phi_1 + 2c_2\Phi_2 + \ldots + nc_n\Phi_n) \right\rangle \rightarrow \left\langle q_1^{c_1} q_2^{c_2} \ldots q_n^{c_n} \cos(c_1\Phi_1 + \ldots + nc_n\Phi_n) \right\rangle / \sqrt{\left\langle q_1^{2c_1} \right\rangle \left\langle q_2^{2c_2} \right\rangle \ldots \left\langle q_n^{2c_n} \right\rangle} \]

- The formula for applying resolution corrections becomes:
  \[
  \left\langle \cos(c_1\Phi_1 + \ldots + nc_n\Phi_n) \right\rangle_{\text{Weighted}} = \frac{\left\langle q_1^{obs,c_1} \times \ldots \times q_n^{obs,c_n} \cos(c_1\Psi_1 + \ldots + nc_n\Psi_n) \right\rangle}{\sqrt{\left\langle (q_1^{obs,P} q_1^{obs,N})^{c_1} \cos(c_1(\Psi_1 - \Psi_1^N)) \right\rangle \ldots \left\langle (q_n^{obs,P} q_n^{obs,N})^{c_n} \cos(nc_n(\Psi_n - \Psi_n^N)) \right\rangle}}
  \]

Resolution for individual planes (2SE method)
Choice of detectors

Subevents used for two-plane correlations and their $\eta$ coverages

<table>
<thead>
<tr>
<th>Subevents used for three-plane correlations and their $\eta$ coverages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Default</strong></td>
<td><strong>ECalFCal$_P$ $\eta \in (0.5,4.8)$</strong></td>
</tr>
<tr>
<td><strong>Cross-check</strong></td>
<td><strong>ID$_P$ $\eta \in (0.5,2.5)$</strong></td>
</tr>
</tbody>
</table>

- **Tracks** $|\eta|<2.5$
- **Inner detector**
- **Default use Calorimeter, ID as cross check**
- **ECal+FCal** $|\eta|<4.9$
Obtaining raw event-plane correlations

Correlation with two planes $\Psi_n, \Psi_m$

<table>
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<tr>
<th>Plane</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_n^N, \Psi_m^N$</td>
<td>ECALFCAL -4.8 &lt; $\eta$ &lt; -0.5</td>
</tr>
<tr>
<td>$\Psi_n^P, \Psi_m^P$</td>
<td>ECALFCAL 0.5 &lt; $\eta$ &lt; 4.8</td>
</tr>
</tbody>
</table>

Each event has two combinations $k(\Psi_n^N - \Psi_m^P)$ and $k(\Psi_n^P - \Psi_m^N)$ with the same resolution.

So just combine into one measurement.

Correlation with three planes $\Psi_n, \Psi_m, \Psi_h$

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<tr>
<td>$\Psi_n^B, \Psi_m^B, \Psi_h^B$</td>
<td>FCAL 3.3 &lt;</td>
</tr>
<tr>
<td>$\Psi_n^C, \Psi_m^C, \Psi_h^C$</td>
<td>ECAL -2.7 &lt; $\eta$ &lt; -0.5</td>
</tr>
<tr>
<td>$\Psi_n^A, \Psi_m^A, \Psi_h^A$</td>
<td>ECAL 0.5 &lt; $\eta$ &lt; 2.7</td>
</tr>
</tbody>
</table>

Each event have $3! = 6$ combinations.
A & C are symmetric so only 3 combined measurements: Type 1,2,3

- Gap is required to remove autocorrelation, more important for Res.
- Event-mixing to check acceptance effect (planes taken from different events)
Expectations from Glauber model

- Plane directions in configuration space
  \[ \epsilon_n = \sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2} \]
  \[ \Phi_n = \tan^{-1} \left( \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle} \right) + \pi \]

- Expected to be strongly modified by medium evolution in the final state (Qiu and Heinz, arXiv:1208.1200)
\[
\frac{\left< \cos 6(\Psi_2 - \Psi_3) \right>}{\text{Res}\left\{6\Psi_2\right\} \text{Res}\left\{6\Psi_3\right\}} = \left< \cos 6(\Phi_2 - \Phi_3) \right>
\]

Small observed signal, good resolution → small corrected signal (<0.02)
Correlation between $\Phi_2$ and $\Phi_4$

- Coefficients decrease slowly with $j$, imply a sharp $\Phi_2$-$\Phi_4$ correlation.
- Very different from correlations in initial state (Glauber)
- What happens if we include final-state-interactions?
Correlation between $\Phi_2$ and $\Phi_4$

- Correlations beautifully reproduced in AMPT model
  - AMPT results from arXiv:1307.0980 (Bhalerao et. al.)
  - Model tuned to reproduce $v_n$ also reproduces EP correlations
Correlation of $\Phi_2$ or $\Phi_3$ with $\Phi_6$

- $\Phi_2$ and $\Phi_3$ weakly correlated, but both strongly correlated with $\Phi_6$.
- They show opposite centrality dependence
  - $\Phi_2$-$\Phi_6$ correlation may due to average geometry..
  - But $\Phi_3$-$\Phi_6$ correlation?
  - $v_6$ dominated by non-linear contribution: $v_2^3$, $v_3^2$?
Correlation of $\Phi_2$ or $\Phi_3$ with $\Phi_6$

- Final state interactions reproduce the correlations
\( \Phi_3 \) vs \( \Phi_4 \) and \( \Phi_2 \) vs \( \Phi_5 \)

\[
\langle \cos 12(\Phi_3 - \Phi_4) \rangle \quad \langle \cos 10(\Phi_2 - \Phi_5) \rangle
\]

correlations are weak (< few %)
Three-plane: "2-3-5" correlation

\[ \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \quad \langle \cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) \rangle \]

- \( \Phi_5 \) and \( \Phi_3 \) are individually weakly correlated with \( \Phi_2 \)
- But \( (2\Phi_2 + 3\Phi_3 - 5\Phi_5) \) correlation is non-zero
- Glauber geometry does not match the correlation
Three-plane: “2-3-5” correlation

\[
\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle
\]

\[
\langle \cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) \rangle
\]

\[
(2\Phi_2 + 3\Phi_3 - 5\Phi_5) = 3(\Phi_3 - \Phi_2) - 5(\Phi_5 - \Phi_2)
\]

\[
(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) = 3(\Phi_3 - \Phi_2) + 5(\Phi_5 - \Phi_2)
\]

- \( \Phi_5 \) and \( \Phi_3 \) are individually weakly correlated with \( \Phi_2 \)
- But \( (2\Phi_2 + 3\Phi_3 - 5\Phi_5) \) correlation is non-zero
- Glauber geometry does not match the correlation
Three-plane: “2-4-6” correlation

\[ \langle \cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6) \rangle \]

\[ \langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle \]
Three-plane: “2-3-4” correlation

\[ \langle \cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4) \rangle \]

\[ \langle \cos(-10\Phi_2 + 6\Phi_3 + 4\Phi_4) \rangle \]
Typically initial geometry in Heavy-Ion collisions is quantified by the eccentricities $\varepsilon_n$:

$$\varepsilon_n e^{i n \Phi_n} \equiv -\frac{\langle r^n e^{i n \phi} \rangle}{\langle r^n \rangle}$$

In a recent paper (arXiv:1206.1905) Teaney and Yan have pointed out that it might be better to quantify the initial geometry by cumulants $c_n$

The cumulants are related to the eccentricities by:

$$c_2 e^{i \Phi_2} \equiv -\frac{\langle z^2 \rangle}{\langle r^2 \rangle}, \quad z = r e^{i \phi}$$

$$c_3 e^{i \Phi_3} \equiv -\frac{\langle z^3 \rangle}{\langle r^3 \rangle},$$

$$c_4 e^{i \Phi_4} \equiv -\frac{1}{\langle r^4 \rangle} \left[\langle z^4 \rangle - 3 \langle z^2 \rangle^2\right],$$

$$c_5 e^{i \Phi_5} \equiv -\frac{1}{\langle r^5 \rangle} \left[\langle z^5 \rangle - 10 \langle z^2 \rangle \langle z^3 \rangle\right],$$

$$c_6 e^{i \Phi_6} = -\frac{1}{\langle r^6 \rangle} \left[\langle z^6 \rangle - 15 \langle z^4 \rangle \langle z^2 \rangle - 10 \langle z^3 \rangle^2 + 30 \langle z^2 \rangle^3\right]$$

Is this parameterization better?
Correlations In initial geometry

Compare correlation between cumulants to the ATLAS EP correlations

1. Do much better job than the correlations between the $\epsilon_n$
2. Indicative that when we define initial geometry in terms of $\epsilon_n$, we have to take into consideration a large degree on non-linear response in generation of the $v_n$

$$v_n e^{i\Phi_n} \propto \epsilon_n e^{i\tilde{\Phi}_n} + \text{significant non-linear contribution from } \epsilon_m (m < n)$$

$$v_n e^{i\Phi_n} \propto c_n e^{i\tilde{\Phi}_n} + \text{small non-linear contribution from } c_m (m < n)$$

See also
arXiv:1312.3689
Teaney & Yan
Can also constrain $\eta/s$, initial geometry

Dependence on $\eta$ gap : EP method

Correlation with two planes $\Psi_n, \Psi_m$
Dependence on $\eta$ gap: SP method

Correlation with two planes $\psi_n^N, \psi_m^N$

$\psi_n^N, \psi_m^N$

Gap

$\psi_n^P, \psi_m^P$

$\langle \cos(\psi_2 - \phi_4) \rangle_w$

$\langle \cos(\psi_6 - \phi_4) \rangle_w$

ATLAS Pb+Pb

$\sqrt{s_{NN}} = 2.76$ TeV

$L_{int} = 7 \, \mu b^{-1}$

The SP method

0-5%

20-25%

40-45%

55-60%
Two-plane correlations: ID
Three-plane correlations: ID

\[ \langle \cos(\Phi) \rangle_w \] vs \( N_{\text{part}} \)

\( \text{ATLAS Pb+Pb} \)
\[ \sqrt{s_{NN}} = 2.76 \text{ TeV} \]
\[ L_{\text{int}} = 7 \mu b^{-1} \]
ATLAS has measured correlations between two and three event planes

- Significant correlations are observed for
  \[ \langle \cos(4(\Phi_2 - \Phi_4)) \rangle, \langle \cos(8(\Phi_2 - \Phi_4)) \rangle, \langle \cos(12(\Phi_2 - \Phi_4)) \rangle, \langle \cos(6(\Phi_2 - \Phi_6)) \rangle, \langle \cos(6(\Phi_3 - \Phi_6)) \rangle \]
  \[ \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle, \langle \cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6) \rangle \]
  and \[ \langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle \]

- Correlation is very small but nonzero for \[ \langle \cos(6(\Phi_2 - \Phi_3)) \rangle \]
- Correlation is negative for \[ \langle \cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4) \rangle \]

- Completely new flow observable

- Most non-zero correlations very different than Glauber model \( \epsilon_n \) correlations.
- Indicate that these are generated dynamically via hydrodynamic evolution.
  
  Qiu and Heinz, arXiv:1208.1200
  
  Teaney and Yan, arXiv:1206.1905

- This measurement provides new constraints for models.
  
  Further constraints on \( \eta/s \), initial geometry

- Indicate that cumulants might be better parameterization of initial geometry